REEXAMINATION OF THE PATCH TEST FOR NONCOMFORMING ELEMENTS

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Nonconforming elements, such as Clough and Wilson do not satisfy the necessary continuity condition at the element boundaries in their variational formulation in linear elasticity. As a result, convergence may not be guaranteed. In order to examine whether or not the finite element approximation converges to the exact solution as mesh size goes to zero even for such nonconforming elements, Irons and Razzaque [1] proposed a brilliant patch test, which assures the convergence without going through a thorough mathematical proof of convergence for a nonconforming element such as Zienkiewicz triangular element for plate bending. This critical study of Irons' patch test was conducted mathematically by Strang [2], who proved that the patch test is a sufficient condition for convergence of nonconforming finite elements, while the research community of the finite element development in engineering tends to consider the patch test being the necessary condition.

Not only Strang but also many other mathematicians challenged to the patch test to provide rigorous mathematical theory, and they have proved that the Wilson element passes the patch test for rectangular and parallelogram shapes. During the course of research, the patch test in engineering is questioned by some mathematicians, see for example Ciarlet [3] who tried to formulate it mathematically. Among such works, Shi [4] revealed that a ruled mesh refinement by the bi-section method assures the convergence of the Wilson quadrilateral, which does not pass the patch test and yet is advocated in engineering field of finite element methods. This paper will make a historical review of the patch test both in engineering and mathematics, and show that the serious of Clough's finite elements, which do not pass the standard patch test in engineering, pass the generalized patch test defined by mathematicians for certain patterns of mesh refinements [5]. In this way, we shall propose reexamination of nonconforming finite elements as a potential for high performance element development, although they may not pass the traditional patch test.

References

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